POWERS OF IDEALS WITH LINEAR OR COMPONENTWISE LINEAR RESOLUTIONS

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Abstract. Let $S = K[x_1, \dots, x_n]$ be a polynomial ring over a field K. For a graded ideal $I \subset S$ we will discuss a criterion in terms of the defining ideal of the Rees algebra of I, which implies that all powers of I have linear quotients and with additional assumptions have componentwise linear resolutions. We will give some applications of this result to various families of monomial ideals arising from combinatorics, establishing that all their powers admit linear or componentwise linear resolutions. Examples include cover ideals of certain graphs and facet ideals of sortable simplicial complexes. As a further application, we explore the complementary version of Herzog-Hibi-Zheng theorem concerning the powers of edge ideals with linear resolutions. More precisely, we show that for any squarefree monomial ideal $I \subset S$ which is generated in degree $n-2$, the following conditions are equivalent.

(i) I has linear resolution.
(ii) I^k has linear resolution, for all $k \geq 1$.
(iii) I^k has linear quotients, for all $k \geq 1$.

We will also discuss a classification problem: for which values of d is the set of squarefree monomial ideals in S with d-linear resolution independent of the base field? We give a complete answer, showing that field-independence occurs precisely for $d \in \{0, 1, 2, n-2, n-1, n\}$.

Finally, we apply the x-condition criterion to the symbolic Rees algebras of certain families of monomial ideals, providing conditions under which all their symbolic powers are componentwise linear.

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